Why Nonlinear Control

M. S. Fadali
Professor of EE

Outline

- Why nonlinear control?
- Classification of nonlinearities.
- Linear systems.
- Examples of nonlinear behavior.

Why Nonlinear Control

1. Real-world systems are nonlinear time-varying.
2. Linearization only valid for a small operational range.
3. Some models cannot be linearized (discontinuous nonlinearities).
4. Robust designs can sometimes be obtained by introducing nonlinearity.
5. Nonlinear feedback can sometimes give simpler controllers.

Classification of Nonlinearities

I. Naturally occurring versus artificially introduced.

II. Continuous versus discontinuous.

I. Natural versus Artificial

Natural

All physical systems for example

Mechanical: Coulomb friction, stiction, square law friction.

Electromagnetic: hysteresis in ϕ-i magnetization curves, relays, saturation.

Electronics: amplifier saturation.

Artificial

Introduced by control system designer

- **Relay in home heating system.**
- **Thrusters in aerospace applications.**
- **Bang-bang control.**
- **Variable structure (switching) control.**

II. Continuous vs. Discontinuous

Continuous

Linearization is possible

$$\mathbf{f}(\mathbf{x}) - \mathbf{f}(\mathbf{x}_0) = \left.\frac{d\mathbf{f}}{d\mathbf{x}}\right]_{\mathbf{x}_0} \Delta\mathbf{x} + O\left(\|\Delta\mathbf{x}\|^2\right)$$

$$\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0$$

$$\Delta\mathbf{f} \approx K\Delta\mathbf{x} \quad \Delta\mathbf{x}\, small$$

Ex. $\quad f = cv^2 \Rightarrow \Delta f = (2cv_0)\Delta v$

Discontinuous ("hard")

Linearization invalid.

Hysteresis

Backlash

Stiction

Linear Systems

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

- Obey the principle of superposition.
- Unique equilibrium point for A nonsingular.

LTI

- Asymptotic stability: LHP eigenvalues.
- Asymptotic stability ⇒ BIBO stability.
- Sinusoidal input ⇒ Sinusoidal output of same frequency.

Examples of Nonlinear Behavior

- Violates superposition.
- Multiple equilibrium points.
- Finite escape time.
- Limit cycles.
- Bifurcation.
- Chaos.