

## Root Locus

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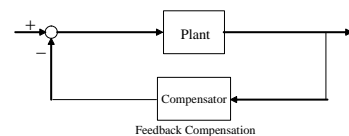
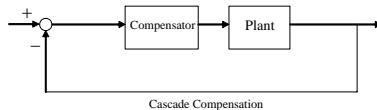
## What is the root locus?

- Plot of the loci of the closed-loop poles of a system as its gain is varied from 0 to  $\infty$ .
- Plots are rarely obtained for negative gains.
- Plots can be obtained for system parameters other than the gain (e.g. time constant).
- Useful design tool since closed-loop poles (and zeros) determine the time response.

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## System Configurations

- In both cases, use the loop gain for root locus plots.
- Only zeros of the forward path are closed-loop zeros.



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## Closed-loop Characteristic Equation

$$1 + KL(s) = 0$$

$$L(s) = \frac{\prod_{i=1}^{n_z} (s - z_i)}{\prod_{j=1}^{n_p} (s - p_j)}$$

- Determined using the loop gain.
- $z_i =$  open-loop zeros,  $p_i =$  open-loop poles

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## Complex Equality

$$KL(s) = -1$$

Equivalent to two real equalities:

i) Magnitude Condition

$$K|L(s)| = 1$$

ii) Angle Condition

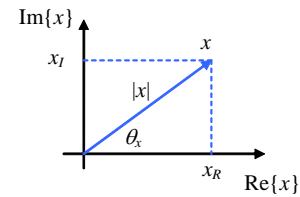
$$\angle L(s) = \pm(2m+1)180^\circ, m = 0, 1, 2, \dots$$

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## Complex Number Arithmetic

$$x = x_R + jx_I \text{ (Cartesian)}$$

$$= |x|e^{j\theta_x} \text{ (polar)}$$



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## Operations

$$x = x_R + jx_I \text{ (Cartesian)}$$

$$= |x|e^{j\theta_x} \text{ (polar)}$$

$$x \pm y = (x_R \pm y_R) + j(x_I \pm y_I)$$

$$x \times y = |x||y|e^{j(\theta_x + \theta_y)}$$

$$x / y = |x|/|y|e^{j(\theta_x - \theta_y)}$$

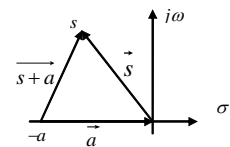
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## Transfer Function

$$G(s) = K \frac{(s+a)}{(s+b)(s+c)}$$

$$|G(s)| = K \frac{|s+a|}{|s+b||s+c|}$$

$$\angle G(s) = \angle(s+a) - \angle(s+b) - \angle(s+c)$$



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### Example: MATLAB

$$G(s) = K \frac{(s+3)(s+4)}{(s+1)(s+2)}$$

- Test Points:  $-2+j3$ ,  $-2+j/\sqrt{2}$  on root locus?
- `g=zpk([-3,-4],[-1,-2],1);`
- `angle(evalfr(g, -2+j*3))`  
 $-1.2315$  ( $-3.1416$  for  $-2+j/\sqrt{2}$ )
- $-1.2315 * 180 / \pi = -70.56$  degrees
- $-2+j3$  not,  $-2+j/\sqrt{2}$  on root locus
- `K=1/abs(evalfr(g,-2+j/sqrt(2)))`

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### Calculator

$$s \leftarrow -2 + i/\sqrt{2}$$

$$g = (s+3) \times (s+4) / (s+1) / (s+2)$$

- Angle:  $-180$  degrees
- On root locus
- Gain:  $K=1/abs(g)=1/3$

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### Example

- Unity feedback closed-loop system with the forward gain:

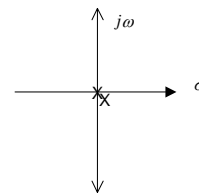
$$G(s) = \frac{K}{s^2}$$

- Closed-loop characteristic equation
- $s^2 + K = 0$ ,  $s_{1,2} = \pm j K^{1/2}$

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### Root Locus for Example 1

$$\text{Root Locus: } \angle G = \pm 90^\circ \pm 90^\circ = \pm 180^\circ$$



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