Phase Margin & Gain Margin

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Stability Margins

- What is the worst perturbation of the transfer function that will make the system marginally stable?
- Marginal stability for open loop stable systems is when the contour goes through the point (–1,0).

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Gain and Phase Margins

Gain Margin: gain perturbation that makes the system marginally stable.

Phase Margin: negative phase perturbation that makes the system marginally stable.

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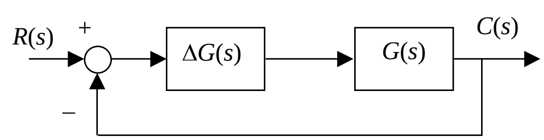
Model Perturbation



- $\Delta G(s)$ = model perturbation
- Gain Margin: $\Delta G(s) = \Delta\, K$ (gain perturbation)
- Phase Margin: $\Delta G(s) = e^{-j\Delta\theta}$ (phase lag perturbation)

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Definitions of Margins

Gain Margin: additional gain that makes the system on the verge of instability.

Phase Margin: additional phase *lag* that makes the system on the verge of instability.

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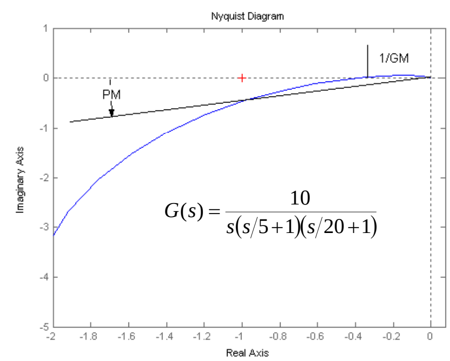
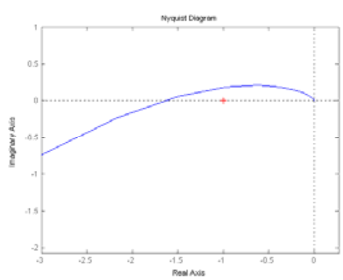
Margins on Polar Plot



$$G(s) = \frac{10}{s(s/5+1)(s/20+1)}$$

Polar plot of Unstable System



Negative GM (dBs) and PM.

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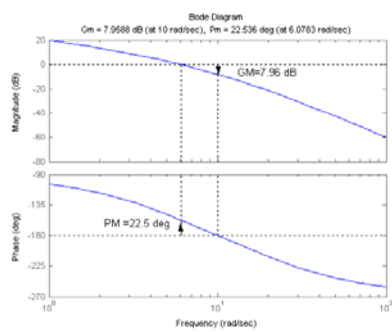
Bode Plot of Stable System

>> margin(g)



MATLAB Margin

```
>> [Gm,Pm,Wcg,Wcp] = margin(g)
Gm =
  2.5000
Pm =
  22.5359
Wcg =
  10.0000 (phase crossover freq.)
Wcp =
  6.0783 (gain crossover freq.)
```

$$G(s)=\frac{10}{s(s/5+1)(s/20+1)}$$

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Gain Margin Calculation

Solve for phase crossover (imaginary part zero)

$$G(j\omega)=\frac{1000}{j\omega(j\omega+5)(j\omega+20)}$$

$$\frac{G(j\omega)}{1000}=\frac{-j(-j\omega+5)(-j\omega+20)}{\omega(\omega^2+25)(\omega^2+400)}$$

$$\mathrm{Im}\{G(j\omega)\}=0 \Leftrightarrow \mathrm{Re}\{(-j\omega+5)(-j\omega+20)\}=0$$

$$\Leftrightarrow 100-\omega^2=0 \Leftrightarrow \omega_{pc}=10 rad/s$$

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Calculate Gain Margin

- Evaluate magnitude at phase crossover frequency

$$GM=-1/G(j10)$$

$$=-\frac{(j10)(j10+5)(j10+20)}{1000}=2.5$$

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Phase Margin Calculation

Solve for gain crossover (unity magnitude)

$$G(j\omega)=\frac{1000}{j\omega(j\omega+5)(j\omega+20)}$$

$$|G(j\omega)|^2=\frac{10^6}{\omega^2(\omega^2+25)(\omega^2+400)}=1$$

$$\Leftrightarrow \omega^6+425\omega^4+10^4\omega^2-10^6=0$$

$$\omega^2=-393.0887,-68.8586,36.9456$$

$$\omega_{gc}=6.078 rad/s$$

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Phase Margin

$$PM = 180^\circ + \angle G(j\omega_{gc}) = 180^\circ + \angle G(j6.078)$$
$$= 180^\circ - 157.4641$$
$$\approx 22.54^\circ$$