Compensator Plane Design for Engineering Models

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Abstract

We characterize value sets and feasible compensation regions for simple plants widely used in engineering applications. We obtain compensation regions that guarantee specified bounds on the control ratio in the compensation plane. We show that for simple plant families the value set is bounded by a subset of the family. We characterize the bounding subset under fixed feedback compensation and show its invariance under fixed cascade compensation. We illustrate the results with a design example. Our results may explain why practicing engineers show little interest in much of the recent developments in control theory.

I. Introduction

There is a growing gap between theory and practice in automatic control due to complex controller designs that often require accurate plant models [12]. In practice, engineers typically have limited knowledge of plants and parameter values. They use simple but adequate plant models and avoid complex mathematical theory and controller structures. Most practicing engineers rely on classical control, modified to exploit computer packages, and seldom use recently developed controller designs

In [12], the authors completely characterize the set of stabilizing PID controllers for first-order systems with time delay. The paper includes a review of similar results for other models and controllers commonly used by practicing engineers. However, it does not address performance robustness for the same practical engineering models. More specifically, the authors do not consider the problem of characterizing controllers that guarantee specified performance robustness for a given transfer function model, which is the main focus of this paper.

We use the methodology of [8], [9], to analyze the design of low order systems with frequency-dependent bounds on the frequency response. We thus obtain a feasible region in the compensation plane. A second feasible region is dictated by compensator structure. The intersection of the two feasible regions, at each frequency, governs the selection of a suitable compensator for the system. Note that we give the feasible regions exactly rather than approximations with specified accuracy as in the work of Nataraj [10], and without the heavy computational burden of gridding approaches [7]. Our work uses geometric concepts akin to those of [6].

For first order or second order plants with simple controllers two or three parameters are available to robustly satisfy design constraints. The plant parameters can be selected to meet design constraints at a critical frequency for the entire family. This yields simple algebraic constraints that can be solved for the controller parameters. Clearly, a more drastic reshaping of the frequency response would require satisfying additional constraints and would only be possible with more complex compensators providing more free parameters.

We show that, for simple families of plants with any fixed controller, frequency response magnitude bounds are obtained from a subset of fixed plants. We prove these results using geometric properties of the inverse value set. These results are valid regardless of design methodology and are an important contribution.

The paper is organized as follows. Section II is a brief review of Nyquist value sets and compensations bounds. Section III gives simple engineering models that are widely used in practical applications. Section IV describes simple practical controllers and their design. Section V examines the value sets and bounds for the models of Section III. Section VI demonstrates the application of the new direct compensation approach to first order systems. Section VII provides conclusions based on the results of earlier sections.

II. Nyquist Value Sets and Compensation Bounds

Consider the closed-loop transfer function (CLTF) $T(s)$:

$$T(s) = \frac{P(s)}{1 + C(s)\cdot P(s)} = \frac{1}{P^{-1}(s) + C(s)} \qquad \textbf{(1)}$$

with plant transfer function $P(s)$ and $P^{-1}(s)$ its reciprocal. We assume that the plant transfer function $P(s)$ is the ratio $A(s,p)/B(s,q)$ of interval polynomials ([2], p. 68), with degrees m and n respectively, $n \geq m$. The coefficients of the polynomials are dependent on physical parameters p and q with known bounds.

To design a satisfactory controller C for the plant transfer function set $P(s)$ in the frequency domain, we require the CLTF of (1) to satisfy magnitude and phase constraints. For example, robust stability gives rise to the RHS while performance constraints give the LHS of

$$M^{-} \leq \left| \left[P^{-1}(j\omega) + C(j\omega)\right|^{-1} \leq M^{+} \qquad \textbf{(2)}$$

A system may be subject to other constraints, such as *tracking bounds*, *sensitivity bounds*, or *actuator bounds*

[5]. A robust design maintains relation (2) for all feasible transfer functions P. We focus on solving for the set $\mathbf{C}=\{C(j\omega),\omega\in\mathscr{R}\}$ satisfying (3)}. Equivalently, a robust design satisfies

$$1/M^+ \le \left| -\left[-P^{-1}(j\omega)\right] + C(j\omega) \right| \le 1/M^- \quad \textbf{(3)}$$

From (3), we observe that, at any given frequency, compensation is allowed where the distance from the plant value set satisfies

$$1/M^+ \le \text{Distance} \le 1/M^- \quad (4)$$

Thus the allowable compensation region for a **fixed** plant transfer function is bounded by two circles centered at $P^{-1}(j\omega)$. For an uncertain plant, compensation regions are given by a series of circles with the center moving along the $P^{-1}(j\omega)$ plot.

III. Practical Engineering Models and Compensators

This section catalogs some simple practical engineering models. These are first-order models, first-order models with a time delay, and second order models. These models may or may not be open-loop stable. Engineers can obtain estimates of the model parameters and bounds on the errors in their estimates. The superscript "+" denotes upper bounds and the superscript "–" denotes the lower bound on a parameter value.

First-order models are of the form

$$P(s)=\frac{K}{Ts+1}, K\in\left[K^-,K^+\right], T\in\left[T^-,T^+\right] \quad \textbf{(5)}$$

where K is the system gain, and T is its time constant.

In some cases, a time delay is added

$$P(s)=\frac{K}{Ts+1}e^{-T_d s}, \quad K\in\left[K^-,K^+\right], T\in\left[T^-,T^+\right], T_d\in\left[T_d^-,T_d^+\right] \quad \textbf{(6)}$$

Another engineering model is the second order system

$$P(s)=\frac{K}{s(Ts+1)}, K\in\left[K^-,K^+\right], T\in\left[T^-,T^+\right] \quad \textbf{(7)}$$

The fourth important model is the second-order system

$$P(s)=\frac{K}{s^2+a_1 s+a_0}, \quad K\in\left[K^-,K^+\right], a_i\in\left[a_i^-,a_i^+,\right], i=0,1 \quad \textbf{(8)}$$

The model (8) is often written in terms of the damping ratio ζ and the angular frequency ω_n.

IV. Practical Controllers

With simple models, engineers typically use simple first-order or second-order compensators. Thus, the CLTF is also relatively low-order and only a few design criteria can be specified. It is therefore possible to obtain the compensator parameter values that meet the design specifications using simple inequalities.

A commonly used control is the proportional-derivative (PD) compensator

$$C(s)=K_p+K_d s \quad \textbf{(9)}$$

PD control improves the transient response. The derivative term is realizable by measuring the derivative of the output. For example, if the output is a position variable an angular or linear velocity sensor provides derivative feedback. The CLTF for a first order plant with PD control is

$$T(s)=\frac{1}{\left(T/K+K_d\right)s+1/K+K_p}, \quad 1/K\in\left[1/K^+,1/K^-\right], \quad T\in\left[T^-,T^+\right] \quad \textbf{(10)}$$

Stability obtains if and only if both denominator coefficients of $T(s)$ are positive, i.e.

$$K_d>-T/K, K_p>-1/K, \quad 1/K\in\left[1/K^+,1/K^-\right], T\in\left[T^-,T^+\right] \quad \textbf{(11)}$$

The stability conditions are always satisfied for open-loop stable systems with a positive gain K. For open-loop unstable systems, higher gains stabilize the system.

For a simple compensator, such as (9), only two parameters are available to meet the design specifications for the entire family. For example, we adopt the DC gain and the bandwidth as the design parameters to choose.

The DC gain of the system is

$$T(0)=\frac{1}{1/K+K_p}, \quad 1/K\in\left[1/K^+,1/K^-\right] \quad \textbf{(12)}$$

We solve for the proportional gain

$$K_p=1/T(0)-1/K, \quad 1/K\in\left[1/K^+,1/K^-\right] \quad \textbf{(13)}$$

To satisfy DC gain constraints

$$M^-(0)<T(0)<M^+(0) \quad \textbf{(14)}$$

we require the proportional gain constraint

$$1/M^+(0)-1/K^+<K_p<1/M^-(0)-1/K^- \quad \textbf{(15)}$$

The bandwidth of the system is given by

$$\omega_{BW}=\frac{1/K+K_p}{T/K+K_d}=\frac{1/T(0)}{T/K+K_d}, \quad 1/K\in\left[1/K^+,1/K^-\right], \quad T\in\left[T^-,T^+\right] \quad \textbf{(16)}$$

We can select K_d to satisfy the bandwidth constraints

$$\omega_{BW}^-<\omega_{BW}<\omega_{BW}^+ \quad \textbf{(17)}$$

The constraints are robustly satisfied with

$$\frac{1}{M^+(0)\omega_{BW}^+}-\frac{T-}{K^+}<K_d<\frac{1}{M^-(0)\omega_{BW}^-}-\frac{T^+}{K^-} \quad \textbf{(18)}$$

Alternatively, select K_p to satisfy (15), then K_d to satisfy

$$\frac{1/K^++K_p}{\omega_{BW}^+}-\frac{T^+}{K^-}<K_d<\frac{1/K^-+K_p}{\omega_{BW}^+}-\frac{T^-}{K^+} \quad \textbf{(19)}$$

Note that the constraints cannot be simultaneously satisfied if either constraint is too tight. The designer must relax the constraints until a feasible solution exists.

For (7) with PD control, the CLTF is in the form

$$T(s)=\frac{1}{(T/K)s^2+(K_d+1/K)s+K_p}$$
$$K\in[K^-,K^+],\qquad T\in[T^-,T^+] \tag{20}$$

Setting $K_p = 1$ makes $T(0)$ unity for all plants in the family. We can then select K_d to obtain the desired bandwidth. The 3 dB point is obtained from

$$K_p-(T/K)\omega^2=(K_d+1/K)\omega$$
$$K\in[K^-,K^+],\qquad T\in[T^-,T^+] \tag{21}$$

We solve for the derivative gain

$$K_d=K_p/\omega-(T/K)\omega-1/K$$
$$K\in[K^-,K^+],\qquad T\in[T^-,T^+] \tag{22}$$

For open-loop stable systems the derivative gain decreases monotonically with frequency. For open-loop unstable systems, the derivative gain has a peak value at

$$\omega_{BW}=\sqrt{KK_p/|T|},K\in[K^-,K^+],T\in[T^-,T^+] \tag{23}$$

If this value is outside the permissible bandwidth range, the derivative gain could either monotonically increase or decrease with frequency. Assuming monotonic decrease and bandwidth governed by the bounds of (17), we select the derivative gain subject to

$$\frac{K_p}{\omega_{BW}^+}-\frac{T^-}{K^+}\omega_{BW}^+-\frac{1}{K^+}<K_d<\frac{K_p}{\omega_{BW}^-}-\frac{T^+}{K}\omega_{BW}^--\frac{1}{K^-} \tag{24}$$

For zero steady-state error due to step and improved transient response, PID control with derivative feedback is a frequently used solution. We use the cascade control

$$C(s)=(K_ps+K_i)/s \tag{25}$$

and the feedback control

$$C_f(s)=K_ds+1 \tag{26}$$

The unity term in $C_f(s)$ guarantees zero steady-state error due to step (see (29)). The CLTF is

$$T(s)=\frac{PC(s)}{1+PCC_f(s)}=\frac{1}{P^{-1}C^{-1}(s)+C_f(s)} \tag{27}$$

The reciprocal of the CLTF is

$$T^{-1}(s)=P^{-1}C^{-1}(s)+C_f(s)$$
$$=\frac{s(Ts+1)/K}{K_ps+K_i}+K_ds+1 \tag{28}$$
$$K\in[K^-,K^+],\qquad T\in[T^-,T^+]$$

The corresponding frequency response is

$$T^{-1}(j\omega)=\frac{j\omega(j\omega T+1)/K}{j\omega K_p+K_i}+j\omega K_d+1$$
$$K\in[K^-,K^+],\qquad T\in[T^-,T^+] \tag{29}$$

We can select a 3 dB frequency at which we impose constraints on the magnitude and phase, or equivalently the real and imaginary parts, of the CLTF at a given frequency. This yields the constraints

$$\operatorname{Re}\{T^{-1}(j\omega)\}=1+\omega^2\frac{K_p-TK_i}{K\left[(\omega K_p)^2+K_i^2\right]}\le\sqrt{2}\cos(\theta)$$
$$\operatorname{Im}\{T^{-1}(j\omega)\}=\omega\left[\frac{-j\omega^2TK_p+K_i}{K\left[(\omega K_p)^2+K_i^2\right]}+K_d\right]\le\sqrt{2}\sin(\theta)$$
$$K\in[K^-,K^+],\qquad T\in[T^-,T^+] \tag{30}$$

The following additional constraint is obtained from the steady-state error due to a ramp

$$K_d\le e_{ramp}(\infty)-\frac{1}{K_iK}$$
$$K\in[K^-,K^+] \tag{31}$$

Numerically solving the three constraints yields the parameters for the PID controller.

V. Simple Value Sets

To design controllers for uncertain linear systems, we examine the value sets of the plants. With the exception of systems with time delay, all value sets for the models of Section III are computed using the general algorithm of [9]. For a bounded time delay T_d, the value set is rotated by an angle ωT_d and [9] can be used with minor changes.

We often need the reciprocal of the value set to determined feasible compensation regions. Table 1 gives some well known reciprocals for simple sets that can be derived from the properties of inversion (see [1]).

Table 1 Reciprocal Value Sets.

Set	Reciprocal Set
Line through origin	Line through origin
Line not through origin	Circle through origin.
Circle through origin	Line not through origin
Circle not through origin	Circle not through origin

Here, we consider the first-order system of (5) in detail. The reciprocal of the system transfer function is

$$P^{-1}(s)=(Ts+1)/K$$
$$1/K\in[1/K^+,1/K^-],\qquad T\in[T^-,T^+] \tag{32}$$

For $s=j\omega$, the value set is shown in Figure 1. The value set is the Minkowski quotient of the two linear value sets corresponding to the two uncertain parameters of the system [9]. Figure 2 shows the permissible compensation region with constraints of the form (2).

Consider the PD control (9). The controller value set is rectangular with width determined by the range of K_p and height by the frequency and the range of K_d. The shaping bounds (2) limit the permissible proportional gains but allow a wide range of derivative gains. The proportional control must satisfy the tightest constraints over all frequencies. The permissible compensation over all frequencies is the dark grey region of Figure 3. The union of the plant value sets is the textured region of Figure 3.

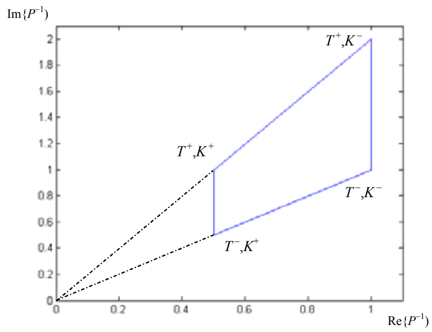

Fig. 1 Reciprocal of value set for first-order plant.

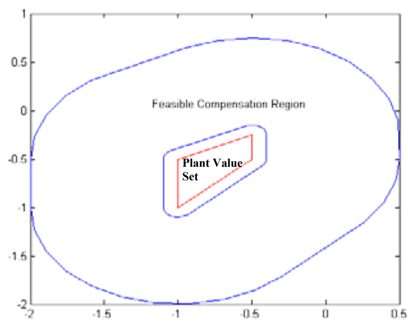

Fig. 2 Feasible compensation region.

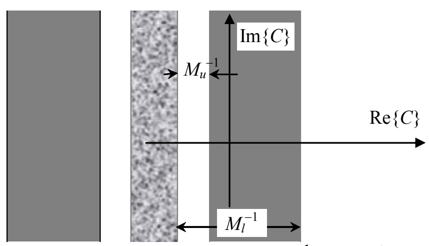

Fig. 3 Compensation regions for P^{-1} for 1st order plant.

V.I Frequency Response Boundary

We often need the boundary of the frequency response set for a family of plants of a given form. For the magnitude, this requires knowledge of the minimum and maximum distance from the value set to the origin. For some cases, the boundary is always formed by a finite set of plants in the family and easily plotted. The following lemmas are useful in this regard. The minimum distance part of the two lemmas was discussed in [10] in the context of stability margin determination.

Lemma 1. The minimum distance from the origin to a line segment is at the intersection with the normal from the origin if it is in the interior of the segment. Otherwise the minimum distance is at a vertex. The maximum distance from the origin is always at a vertex.

Lemma 2. The minimum distance from the origin to the reciprocal of a straight line segment (circular arc) is at a vertex. The maximum distance is on a diameter if a point on the diameter is in the interior of the circular arc. Otherwise the maximum distance is at a vertex.

Proof: Both lemmas follow from the triangle inequality. ♦

Lemma 3. If a line segment or circular arc is multiplied by a fixed complex number, the point of minimum (maximum) distance from the origin of the transformed line segment or circular arc is obtained by transforming the point of minimum (maximum) distance. If a fixed complex number is added to a line segment or circular arc, the minimum (maximum) is governed by Lemma 1 for a line segment and by Lemma 2 for a circular arc.

Proof: Addition transforms a line segment (circular arc) to another line segment (respectively, another circular arc). Multiplication is equivalent to rotation, which has no effect on magnitude, and scaling the entire set, which does not change relative magnitude. ♦

Clearly, the minimum and maximum phase for a line segment, or for a circular arc that is the reciprocal of a line segment, are both at a vertex. This provides a starting point for an optimal algorithm to calculate the Minkowski quotient of two interval polynomials derived in [9]. The quotient is obtained by determining the boundary of sixteen circular arcs and sixteen line segments. ♦

Theorem 1. For uncertain interval systems

(i) The minimum frequency response at a given frequency is obtained by searching the set of vertices of its sixteen circular arcs and sixteen line segments, as well as those intersections with normals from the origin that lie in the interior of a line segment.

(ii) The maximum frequency response at a given frequency is obtained by searching over the set of vertices of its sixteen circular arcs and sixteen line segments, as well as those intersections with points on diameters that lie in the interior of a circular arc.

Proof: The results follow from Lemmas 1 and 2. ♦

Theorem 1 allows determining the frequency response boundary of an interval family of transfer functions. Unfortunately, it requires elaborate computation because the bounding transfer function changes with frequency. Fortunately, for many practical transfer functions, the boundary is at a few known systems.

Theorem 2. For the uncertain systems (5), (6), or (7), the minimum frequency response magnitude and angle occur at the vertex corresponding to (T^+, K^-) and the maximum occur at the vertex corresponding to (T^-, K^+).

Proof: The reciprocal of the value set for (5) is shown in Figure 1. Lemma 1 implies that the maximum and minimum distance from the origin is at a vertex. Figure 1 shows that, at any frequency, the maximum is at (T^+, K^-) and the minimum occurs at (T^-, K^+). The maximum and minimum are interchanged for the value set as opposed to its reciprocal. By Lemma 3, the minimum and maximum are unchanged if the value set is multiplied by a fixed complex number. The reciprocal value set of (7) is that of (5) multiplied by the fixed value $j\omega$. Further, the relative angle magnitudes are unchanged by $j\omega$. For (6), we obtain the value set by rotating the value set of (5), which has no effect on magnitudes and relative angles. ♦

Corollary 1. For uncertain open-loop stable system of (8), the minimum for the frequency response at any frequency occurs at one of the two vertices

$$P(s)=\frac{K^{+}}{s^{2}+a_{1}^{+}s+a_{0}},\qquad a_{0}\in\left\{a_{0}^{-},a_{0}^{+}\right\} \tag{33}$$

The maximum occurs on the edge of the value set

$$P(s)=\frac{K^{+}}{s^{2}+a_{1}^{-}s+a_{0}},\qquad a_{0}\in\left[a_{0}^{-},a_{0}^{+}\right] \tag{34}$$

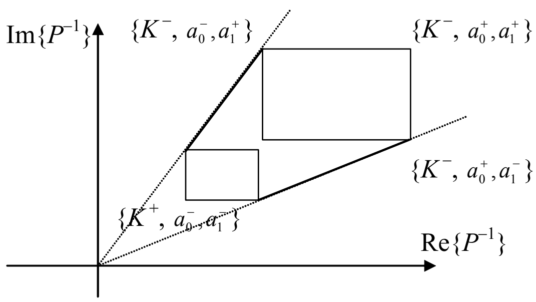

Fig. 4 Value set for 2nd order system.

Proof: For the system of (8), the reciprocal value set is a swept rectangle lying in the first and second quadrants (Figure 4). The reciprocal value set is bounded by six line segments. The maximum distance from the origin (minimum of the value set) is at one of the two vertices $\{K^{-},\ a_0^{-},a_1^{+}\}$ and $\{K^{-},\ a_0^{+},a_1^{+}\}$. The minimum distance, which corresponds to the maximum of the value set, is on the edge of the value set closest to the origin. ♦

Theorem 2 gives frequency response bounds for (5). In addition, The minimum distance occurs at frequencies in the interval $\omega^{2}\in\left[a_0^{-},a_0^{+}\right]$. Theorem 3 yields bounds with fixed compensation.

Theorem 3. For an uncertain plant whose value sets are bounded by line segments and circular arcs, with fixed cascade compensation, the parameter values at which the maximum (minimum) frequency response occurs are the same as the uncompensated plant. For fixed feedback compensation, the maximum distance is at a vertex or on a diameter of an arc, while the minimum is at a vertex or at the intersection of a normal to a line segment.

Proof: The reciprocal value set for a system with cascade and feedback compensation is given by (28). Using Lemmas 1 and 2, the reciprocal value set is obtained by multiplying the reciprocal value set of the plant by a complex number (reciprocal of the feedback compensation) then adding a complex number (cascade compensation). By Lemma 3, this does not change the location of the maximum and minimum. ♦

Using Theorem 3, we can now easily determine the boundary of the value set for any system of the form (5)-(7). Corollary 2 requires gridding the family of (34).

VI. Design Example

We examine PI and PID control of a velocity control system consisting of a DC motor and load. The plant model is of the form

$$P(s)=\frac{[1.5,2]}{[0.9,1]s+1}$$

We require a DC gain $K\in[0.9,\ 1.1]$ and a bandwidth $\omega_b\approx 2$ rad/s. The feasible compensation region for the system is as shown in Figure 2 for a frequency $\omega=1$ rad/s. For a DC gain in the range [0.9,1.1], (15) and (18) yield

$$0.4091<K_p<0.4444$$

$$0.9091/\omega_{BW}^{+}-0.45<K_d<1.1111/\omega_{BW}^{-}-0.6666$$

For a bandwidth of 2 rad/s, we have the constraints

$$0.004545<K_d<0.0740740$$

We select the compensator

$$C(s)=0.042(s+10)$$

For a PD compensator, the CLTF is of the form (5). By Theorem 2, frequency responses for the family are bounded by the frequency responses of the four vertices, shown in Figure 5. The time responses of the system for parameter values corresponding to the vertices of the value set are shown in Figure 6. The time responses are faster than those of the open-loop system.

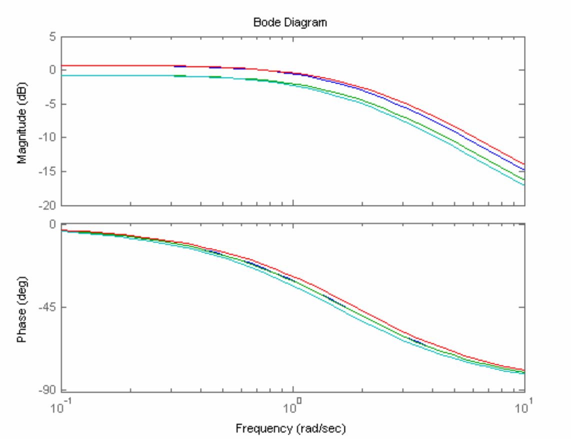

Fig. 5 Frequency response boundary of PD-controlled 1st order system.

A typical design for the first order system often yields an excellent time response for a nominal model but fails to guarantee performance robustness for the family of models considered in this example. We obtain acceptable robust performance for all plants of the family at the cost of some deterioration in the behavior of the nominal plant. Note that for a DC gain $K\in[0.95,1.05]$ there is no solution that satisfies the constraints (15) and (18).

For PID control with derivative feedback, we select a bandwidth of about 2.5 rad/s and an angle of $2\pi/3$. We require the steady-state error due to unit ramp to be less than one third. The constraints yield the cascade control

$$C(s)=\left(0.192864\times10^{-3}s+2.036155\right)/s$$

and the feedback control

$C_f(s) = 0.365212s + 1$

By Theorem 2, the frequency responses at the vertices of the value set give bounds on the closed-loop frequency response (Figure 7). The corresponding time responses of Figure 8 demonstrate the robustness of the design.

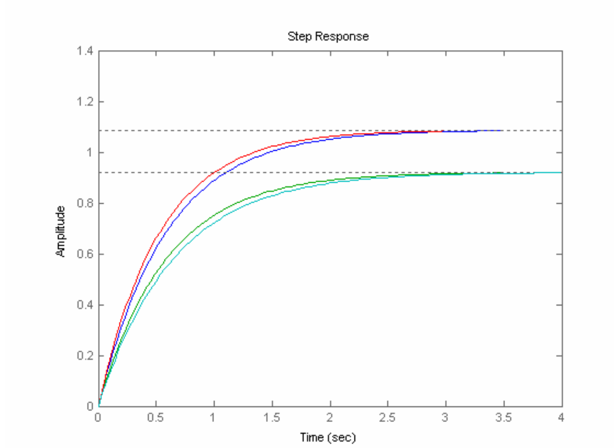

Fig. 6 Step response of PD-controlled 1st order system.

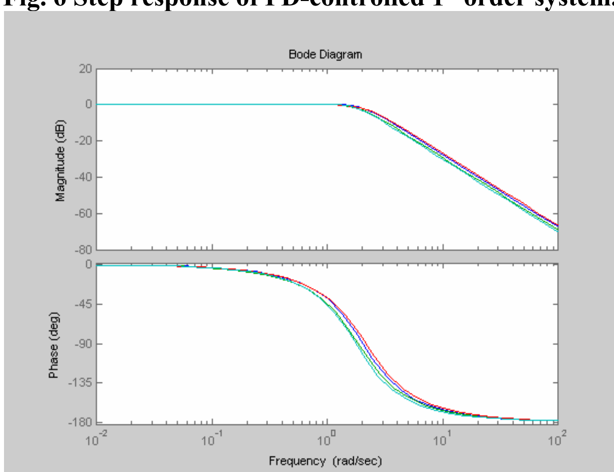

Fig. 7 Frequency response boundary of PID-controlled 1st order system.

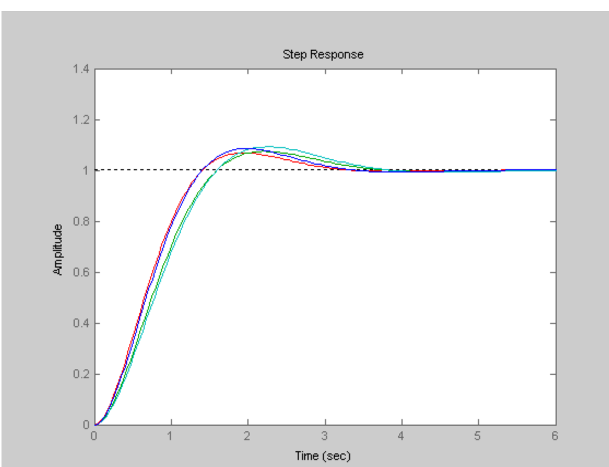

Fig. 8 Step response of PID-controlled 1st order system.

VII. Conclusion

Practical design methodologies must be (i) robust with respect to modeling errors, and (ii) suited to the simple models that are used in the majority of engineering applications. Furthermore, they must yield simple first or second order controllers that are easily implemented and tested. By restricting our analysis to simple practical models, we provide easily computed exact bounds on the frequency response for first and second order plants. We show that designing simple controllers for such plants only allows the choice of a few design parameters. Both design and bound computation do not require advanced mathematical theory that is beyond the capabilities and inclinations of most practicing engineers. By contrast much of the recent control literature emphasizes highly mathematical methodologies that yield high-order controllers [4]. Such controllers have great theoretical value but are unlikely to interest the vast majority of practicing engineers. Thus, the results of this paper provide reasons for the gap between control theory and control practice.

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