A New Approach for Designing TSK Fuzzy Systems From Input-Output Data

Assem Sonbol	M. Sami Fadali
Electrical Engineering/260	Electrical Engineering/260
University of Nevada	University of Nevada
Reno, NV 89557	Reno, NV 89557
sonbol@unr.edu	fadali@ieee.org

Abstract

We propose a new approach for determining fuzzy membership functions for system inputs and outputs and for defining the rule base systematically from input-output pairs. This approach treats a human-operated system as a black box and does not require knowledge of its underlying mathematical model. We provide an upper bound on the error resulting from approximating the input-output data using a fuzzy system. To demonstrate the ease and power of the new approach, we apply it to the benchmark problem of backing up a truck into a loading dock. The results show that the fuzzy controller can be chosen as a compromise between the conflicting requirements of tracking accuracy and simplicity of implementation.

I. Introduction

Fuzzy systems can be used to model human knowledge in engineering problems. This knowledge may be classified into two categories: (i) conscious knowledge explicitly expressible in words, and (ii) subconscious knowledge that a human expert translates into actions but cannot explain in words.

We can express conscious knowledge in terms of fuzzy IF-THEN rules and implement the rules in fuzzy systems. For subconscious knowledge, we can view the human expert as a black box and measure its relevant inputs and outputs. Thus, we represent subconscious knowledge by a set of input-output pairs. Hence, a problem of fundamental importance is to construct fuzzy systems from input-output pairs. To solve this problem, we need to (i) determine the membership functions of the fuzzy input and output sets, and (ii) define a fuzzy rule base.

Some of the most significant applications of fuzzy theory have concentrated on control problems where human operators, who intuitively know the behavior of the system, provide the best control. The human operator may not always be able to satisfactorily control a process but can usually prescribe a fairly good recipe for control in familiar situations. This recipe, or control strategy, may take the form of a set of situation-action pairs (rules), known to the human operator, around which the control system can be designed. This provides the core knowledge for the system, but usually does not completely describe it. To complete this description, we must define fuzzy sets representing the range of control parameters or sensor fields. The system description is then finely tuned by adjusting the fuzzy sets and the rule base.

Many researchers have developed approaches to determine fuzzy sets and rule bases. For example, Manoranjan *et al.* [5] proposed a systematic approach for determining the grades of membership of fuzzy subsets that represent the behavior of a control practitioner. Abe and Lan [1] developed a method for extracting fuzzy rules directly from numerical input-output data for pattern classification and function approximation. Homaifar and McCormick [2] examined the applicability of genetic algorithms in the simultaneous design of membership functions and rule sets for fuzzy logic controllers. Ishibuchi *et al.* [3] used genetic algorithms for selecting a small number of significant fuzzy IF-THEN rules to construct a compact fuzzy classification system with high classification power. Perneel *et al.* [6] used genetic algorithms and neural networks as optimization methods to improve the performance of the fuzzy decision-making system for heuristic search algorithms. Rovatti *et al.* [7] proposed a new methodology for the minimization of a given set of fuzzy rules. Rovatti and Guerrieri [8] also developed an on-line adaptive algorithm, which learned the extent to which inclusion of a rule in the rule set significantly contributed to the reproduction of the target behavior for system identification.

This paper introduces a new “black box” approach to fuzzy control for systems with no known mathematical model but with measurable inputs and outputs. Such systems are common in engineering applications and cannot be analyzed using conventional control methodologies. We show how the input-output data can be used to obtain the membership functions needed for fuzzy control. The membership functions are selected to optimize an approximation of the truncated Taylor series expansion of the unknown mathematical model of the fuzzy controller. We provide a bound on the approximation error when using the fuzzy model.

The paper is organized as follows: In Section II, we introduce some basic concepts and notations of fuzzy sets. In Section III, we give a mathematical analysis of the Takagi-Sugeno-Kang (TSK) fuzzy system. In Section IV, we present our new approach for designing fuzzy systems; that is, our method for determining fuzzy sets for the input variables and building the fuzzy rule base. In Section V, we apply the new approach to the well known truck backer-upper system. In Section VI, we give simulation results for the truck backer-upper under different initial conditions. We discuss the results and compare them with results from the literature. Conclusions and suggestions for future work are given in Section VII.

II. Properties of Fuzzy Sets

A fuzzy set A is characterized by a membership function $\mu_A(x)$ with values in the unit interval [0,1] and x in a known universe of discourse U. We summarize some properties of fuzzy sets that are adopted by most researchers (see, for example, [10]).

Definition 1: Completeness

Fuzzy sets $A^i, i = 1, \cdots, N,$ in $U \subset R$ are said to be a complete partition on U if for any x ∈ U, ∃ at least one fuzzy set A^j such that $\mu_{A^j}(x) > 0$.

A set of rules is complete if at any point in the input space there is at least one rule for which the membership value of the antecedent of the rule is non-zero [10].

Definition 2: Consistency

Fuzzy sets $A^i, i = 1, \cdots, N,$ in $U \subset R$ are said to be consistent on U if ∃ $x \in U$ such that $\mu_{A^j}(x) = 1$ implies $\mu_{A^j}(x) = 0$ for all $i \neq j$.

A set of fuzzy IF-THEN rules is consistent if all rules with the same antecedent have the same consequent.

Definition 3: Continuity

A set of fuzzy IF-THEN rules is continuous if there are no neighboring rules whose consequent fuzzy sets have empty intersection.

Continuity guarantees a smooth input-output behavior for the fuzzy system.

Definition 4: Normality

A set of fuzzy IF-THEN rules is normal if the largest membership value, known as the height of the fuzzy set, is unity.

III. Mathematical Analysis of the TSK System

In this section, we analyze the TSK fuzzy multi-input multi-output (MIMO) system. Because a MIMO system can be viewed as a set of multi-input single-output (MISO) fuzzy system, without loss of generality, we assume that fuzzy systems are MISO mappings $f: U \subset R^n \rightarrow V \subset R$ where $U = U_1 \times U_2 \times \ldots \times U_n \subset R^n$ is the input space and $V \subset R$ is the output space [9]. The TSK model consists of four principal components:

1. A fuzzifier that consists of normal, complete and consistent fuzzy sets.
2. A complete fuzzy rule base of the form:

$$R_{i_1 i_2 \ldots i_n} : IF \quad \bigcap_{j=1}^{n} x_j \quad is \quad A_j^{i_j} \quad THEN \quad y = C_{i_1 i_2 \ldots i_n} \qquad (1)$$
$$i_j = 1,2,\ldots, N_j, \quad j = 1,2,\ldots,n$$

   where x_j, j = 1, 2, …, n, are the input variables of the fuzzy system, y is its output variable, and N_j is the number of membership functions in the j^{th} input. The fuzzy sets $A_j^{i_j} \subset U_j$, i_j=1,2,…,N_j and j=1,2,…,n, are linguistic terms characterized by fuzzy membership functions $\mu_{A_j^{i_j}}(x_j)$. $C_{i_1 i_2 \ldots i_n} \subset V$ is a constant corresponding to the rule $R_{i_1 i_2 \ldots i_n}$.

3. A fuzzy inference engine with the T-norm in fuzzy implication and product inference.
4. A weighted-average defuzzifier.

Lemma 1: The TSK fuzzy approximation of a nonlinear MISO system with triangular membership functions can be considered as a truncated Taylor series expansion of its input-output mapping.

Proof: We prove the result for the 2-input case to avoid cumbersome notation. Generalization to multi-input is straightforward. In the above fuzzy system, at least one and at most two membership functions $\mu_{A_j^{i_j}}(x_j)$ are nonzero for i_j=1,2,…,N_j and j=1,2,…,n.

These two possible nonzero functions are $\mu_{A_j^{i_j}}(x_j)$ and $\mu_{A_j^{i_j+1}}(x_j)$. Hence, the fuzzy system can be written as follows [10]

$$f(\mathbf{x}) = \frac{\sum_{m_1=i_1}^{i_1+1} \cdots \sum_{m_n=i_n}^{i_n+1} (\prod_{j=1}^{n} \mu_{A_j^{m_j}}(x_j)) C_{m_1 \dots m_n}}{\sum_{m_1=i_1}^{i_1+1} \cdots \sum_{m_n=i_n}^{i_n+1} (\prod_{j=1}^{n} \mu_{A_j^{m_j}}(x_j))} \qquad (2)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)^T \in U_1 \times U_2 \times \dots \times U_n \subset R^n$ and $C_{i_1 i_2 \dots i_n} \in V \subset R$, $i_1\ i_2\ \dots\ i_n \in I$. Because of the normality and consistency of the fuzzy sets, the denominator in (2) is unity and we can rewrite (2) as:

$$f(\mathbf{x}) = \sum_{m_1=i_1}^{i_1+1} \cdots \sum_{m_n=i_n}^{i_n+1} (\prod_{j=1}^{n} \mu_{A_j^{m_j}}(x_j)) C_{m_1 \dots m_n} \qquad (3)$$

By consistency of the fuzzy sets, we have

$$\mu_{A_j^{i_j}}(x_j) + \mu_{A_j^{i_j+1}}(x_j) = 1 \qquad (4)$$

If the system is two-input-single-output, the fuzzy approximation can be written as:

$$\begin{aligned} f(\mathbf{x}) &= C_{i_1 i_2} + (C_{i_1+1, i_2} - C_{i_1 i_2}) \mu_{A_1^{i_1+1}}(x_1) \\ &\quad + (C_{i_1, i_2+1} - C_{i_1 i_2}) \mu_{A_2^{i_2+1}}(x_2) \\ &\quad + \left(C_{i_1 i_2} - C_{i_1, i_2+1} - C_{i_1+1, i_2} + C_{i_1+1, i_2+1}\right) \mu_{A_1^{i_1+1}}(x_1) \mu_{A_2^{i_2+1}}(x_2) \end{aligned} \qquad (5)$$

From Figure1, we obtain the membership functions

$$\begin{aligned} \mu_{A_1^{i_1+1}}(x_1) &= \frac{1}{h_{i_1, i_1+1}} (x_1 - x_0^{i_1+1}) \\ \mu_{A_2^{i_1+1}}(x_2) &= \frac{1}{h_{i_2, i_2+1}} (x_2 - x_0^{i_2+1}) \end{aligned} \qquad (6)$$

where $x_0^{i_j+1} \le x_j \le x_1^{i_j+1}, h_{i_j, i_j+1} = x_1^{i_j+1} - x_0^{i_j+1}$, and $\left(x_0^{i_j+1}, x_1^{i_j+1}\right)$ are the points where the membership function values of $A_j^{i_j+1}$ are (0,1) respectively.

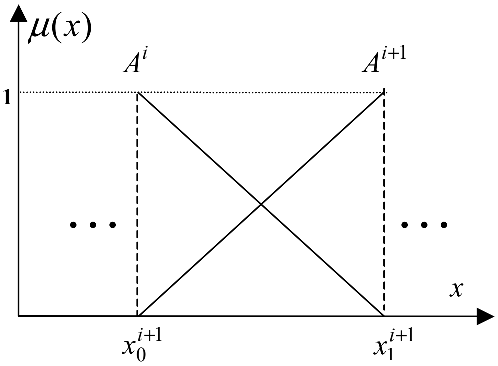

Figure 1: *The overlap between two triangular Membership functions.*

Substituting from (6) into (5)

$$\begin{aligned} f(\mathbf{x}) &= f(x_0^{i_1+1}, x_0^{i_2+1}) \\ &+ \frac{\left(f(x_1^{i_1+1}, x_0^{i_2+1}) - f(x_0^{i_1+1}, x_0^{i_2+1})\right)}{h_{i_1, i_1+1}} \Delta x_1 \\ &+ \frac{\left(f(x_0^{i_1+1}, x_1^{i_2+1}) - f(x_0^{i_1+1}, x_0^{i_2+1})\right)}{h_{i_2, i_2+1}} \Delta x_2 \\ &+ \frac{\left(f(x_0^{i_1+1}, x_0^{i_2+1}) - f(x_1^{i_1+1}, x_0^{i_2+1})\right)}{h_{i_1, i_1+1} h_{i_2, i_2+1}} \Delta x_1 \Delta x_2 \\ &- \frac{\left(f(x_0^{i_1+1}, x_1^{i_2+1}) - f(x_1^{i_1+1}, x_1^{i_2+1})\right)}{h_{i_1, i_1+1} h_{i_2, i_2+1}} \Delta x_1 \Delta x_2 \end{aligned} \qquad (7)$$

where $\Delta x_1 = (x_1 - x_0^{i_1+1})$, and $\Delta x_2 = (x_2 - x_0^{i_2+1})$.

The second order approximation of the input-output relation is

$$\begin{aligned} f_T(\mathbf{x}) &= f(x_0^{i_1+1}, x_0^{i_2+1}) \\ &+ \left.\frac{\partial f}{\partial x_1}\right|_{\substack{x_1 = x_0^{i_1+1} \\ x_2 = x_0^{i_2+1}}} .\Delta x_1 + \left.\frac{\partial f}{\partial x_2}\right|_{\substack{x_1 = x_0^{i_1+1} \\ x_2 = x_0^{i_2+1}}} .\Delta x_2 \\ &+ \frac{1}{2!} \left(\left.\frac{\partial^2 f}{\partial x_1^2}\right|_{\substack{x_1 = x_0^{i_1+1} \\ x_2 = x_0^{i_2+1}}} .\Delta x_1^2 + \left.\frac{\partial^2 f}{\partial x_2^2}\right|_{\substack{x_1 = x_0^{i_1+1} \\ x_2 = x_0^{i_2+1}}} .\Delta x_2^2 \right) \\ &+ \frac{1}{2!} \left(2 \left.\frac{\partial^2 f}{\partial x_1 \partial x_2}\right|_{\substack{x_1 = x_0^{i_1+1} \\ x_2 = x_0^{i_2+1}}} .\Delta x_1 \Delta x_2 \right) + O\left(\|\Delta \mathbf{x}\|^2\right) \end{aligned} \qquad (8)$$

Assuming constant slopes $\partial f / \partial x_j$ over the intervals $(x_0^{i_1+1}, x_1^{i_1+1})$ and $(x_0^{i_2+1}, x_1^{i_2+1})$ respectively, then $\partial^2 f / \partial x_j^2 = 0$, $j = 1,2$, and we can rewrite (8) as

$$f_T(\mathbf{x}) = f(\mathbf{x}) \qquad (9)$$

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We now give a bound on the approximation error for the TSK fuzzy system based on Lemma 1.

Theorem 1: The first order approximation error of the TSK fuzzy system of Lemma 1 is bounded by

(i) $\left|g(\mathbf{x}) - f(\mathbf{x})\right| \le \left\|\varepsilon_{\mathbf{x}}\right\|_{\infty} . \left\|h_{\mathbf{x}}\right\|_{1}$ (10a)

(ii) $\left|g(\mathbf{x}) - f(\mathbf{x})\right| \le \left\|\varepsilon_{\mathbf{x}}\right\|_{2} \left\|h_{\mathbf{x}}\right\|_{2}$ (10b)

where, $g(\mathbf{x})$ is the response of the actual system, $f(\mathbf{x})$ is the response of the fuzzy system, ε_{x_i} is the maximum error in $\partial f / \partial x_i$, h_{x_i} is the maximum overlap width in x_i membership functions, i =1,2, $\varepsilon_{\mathbf{x}} = [\varepsilon_{x_1} \; \ldots \; \varepsilon_{x_n}]$, $\left\|\cdot\right\|_2$, and $\left\|\cdot\right\|_{\infty}$ are the 2-norm and the infinity-norm respectively.

Proof: We assume that the value of $g(\mathbf{x}_0^{\mathbf{i}+1})$, where $\mathbf{x}_0^{\mathbf{i}+1} = \left[x_0^{i_j+1}\right]_{j=1,2,\cdots,n}$, is known exactly. Hence,

$$g(\mathbf{x}) - f(\mathbf{x}) = \left(\frac{\partial g}{\partial \mathbf{x}} - \frac{\partial f}{\partial \mathbf{x}}\right)^T \Delta\mathbf{x} + O\left(\left\|\Delta\mathbf{x}\right\|^2\right) \qquad (11)$$

$$\left|g(\mathbf{x}) - f(\mathbf{x})\right| \le \left|\left(\frac{\partial g}{\partial \mathbf{x}} - \frac{\partial f}{\partial \mathbf{x}}\right)^T \Delta\mathbf{x}\right| + O\left(\left\|\Delta\mathbf{x}\right\|^2\right)$$
$$\le \left\|\left(\frac{\partial g}{\partial \mathbf{x}} - \frac{\partial f}{\partial \mathbf{x}}\right)\right\|_{\infty} \left\|\Delta\mathbf{x}\right\|_1 + O\left(\left\|\Delta\mathbf{x}\right\|^2\right) \qquad (12)$$

Given that

$$\left|\frac{\partial g}{\partial x_i} - \frac{\partial f}{\partial x_i}\right| \le \left\|\varepsilon_{\mathbf{x}}\right\|_{\infty}, \; i = 1,2,\ldots,n$$
$$\Delta x_j \le \max_{i_j} h_{i_j, i_j+1} = h_{x_j} \qquad (13)$$

Substituting (13) into (12), we get the first order approximation

$$\left|g(\mathbf{x}) - f(\mathbf{x})\right| \le \left\|\varepsilon_{\mathbf{x}}\right\|_{\infty} . \left\|h_{\mathbf{x}}\right\|_{1}$$

where $h_{\mathbf{x}} = \left(h_{x_1}, h_{x_2}, \ldots, h_{x_n}\right)^T$ and $\left\|h_{\mathbf{x}}\right\|_1 = \sum_{i=1}^{n} h_{x_i}$

Alternatively, we can use the 2-norm to obtain

$$\left|g(\mathbf{x}) - f(\mathbf{x})\right| \le \left\|\left(\frac{\partial g}{\partial \mathbf{x}} - \frac{\partial f}{\partial \mathbf{x}}\right)\right\|_{2} \left\|\Delta\mathbf{x}\right\|_2 \qquad (14)$$

Then,

$$\left|g(\mathbf{x}) - f(\mathbf{x})\right| \le \left\|\varepsilon_{\mathbf{x}}\right\|_{2} \left\|h_{\mathbf{x}}\right\|_{2} \qquad (15)$$

•

Remark: Numerical values determine which of the two bounds of (10) is tighter.

IV. The New Approach

Assuming that the human operated system to be approximated using a fuzzy system has an unknown mathematical model $g(.)$ such that the output $g(\mathbf{x})$ can be determined for any $\mathbf{x} \in U$. Then, based on equations (1)-(12), together with our previous conclusions, we need to collect a set of input-output pairs that represent a system trajectory from any initial point to a known steady state. From this set, we compute piecewise linear approximations of $\partial g / \partial x_i, i = 1, \cdots, n,$ and use them to obtain the fuzzy membership function for each input (see Figure 2, for example). Then, the fuzzy rule base with $M_1 \times M_2 \times \ldots \times M_n$ rules of the form (1) can be easily constructed to determine $C_{i_1 i_2 \cdots i_n}$ for each rule, where M_j is the number of fuzzy sets for the j^{th} input.

Using Theorem 1, we can obtain the number of membership functions necessary to achieve a specified upper bound on the error involved in approximating the measured input-output behavior with a TSK system. Reducing the error requires additional membership functions, i.e. a more complex fuzzy system.

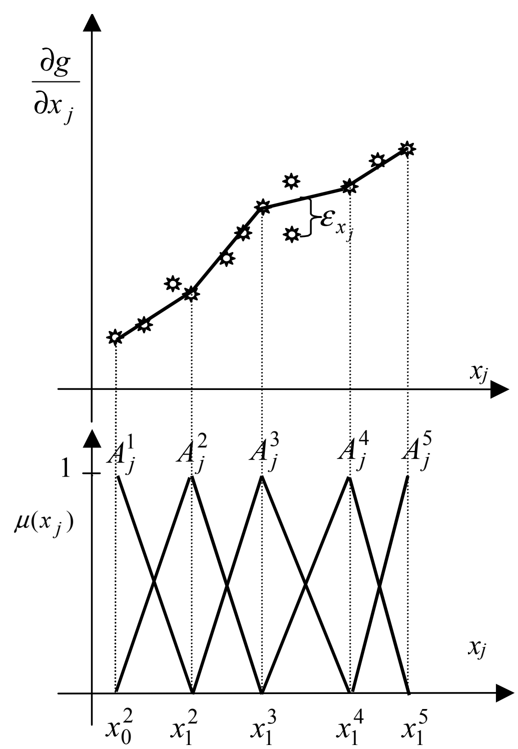

Figure 2 D*etermination of fuzzy sets from input-output data (* ✲ *= data points).*

V. Application of the New Approach

We use the problem of backing up a truck to a loading dock to show how to apply the newly proposed approach to design a fuzzy system to mimic the human driver. We assume that an experienced human driver is available and that we can measure the truck's states and the corresponding control action of the human driver while backing the truck into the dock; that is, we can collect sets of input-output pairs. Then, we can design a fuzzy system to replace the human driver based on these input-output pairs.

Problem Description: The truck position is determined by three state variables ϕ, x, and y, where ϕ is the angle of the truck with respect to the horizontal line. The coordinate pair (x, y) specifies the position of the rear center of the truck in the plane. Control to the truck is the steering angle θ that backs up the truck to the loading dock from any initial position and from any angle in the loading zone. Only backing up is permitted. The truck moves backward by a fixed unit distance every stage. The goal is to make the truck arrive at the loading dock at a right angle ($\phi = 90°$) and to align the position (x, y) of the truck with the desired loading dock (x_f, y_f). The loading zone corresponded to the plane $[0,100]\times[0,100]$, and (x_f, y_f) equal to (50, 100). For simplicity, we assume enough clearance between the truck and the loading dock such that y does not have to be considered as a state variable [4], [10]. We use the fuzzy system described in [4] to represent the human action.

VI. Experimental Results

The simulated response of the human operated system for different initial conditions is shown in Figures 3 and 4. The corresponding response of our fuzzy system for the same initial conditions is shown in Figures 5 and 6. The fuzzy system can successfully back up the truck into the loading dock in every case but does not exactly match the trajectories obtained for the human controller. Increasing the number of membership function used in the TSK approximation reduces the tracking error but complicates the implementation of the fuzzy controller considerably. The number of membership functions used must be a compromise between tracking accuracy and ease of implementation. We used seven input and seven output triangular membership functions to obtain the trajectories of Figures 5 and 6.

To assess the performance of the proposed controller, we compare it to two other controllers from the literature: a controller designed using genetic algorithms (GA) [2], and a controller designed using input-output pairs [11]. Homaifar and McCormick [2] used a 7-5-7 controller (7 membership functions for x, 5 for ϕ, and 7 for θ). They chose the number of membership sets of the input and output variables and consequently the number of fuzzy rules. Thereafter, they used a GA to design the input membership functions and the rule base, keeping the output membership functions constant. Wang and Mendel [11] developed a method that first chooses the number of membership sets for the inputs and the output, then generates the fuzzy rules using the input-output pairs that represent some successful trajectories. Thus, they needed a sufficiently large number of successful trajectories to construct a continuous fuzzy rule base.

By contrast, our approach designs the entire controller (input and output membership functions including their numbers and set widths together with the fuzzy rule base) using one exemplary trajectory. It also avoids difficulties that can arise during the application of GA such as choosing the suitable fitness function, string length, number of alleles per generation, number of generations, and excessive computational time. In addition, we use a reasonable predetermined number of input-output pairs in generating the fuzzy rules. This makes our approach better than that of [11] and better than neural networks that use a huge number of input-output pairs without any guaranteed convergence.

VII. Conclusion and Further work

This paper proposes a new approach for designing fuzzy systems from input-output data pairs. We show that a TSK fuzzy controller with triangular membership functions is equivalent to a truncated Taylor series expansion of a nonlinear function. We obtain an upper bound on the approximation error that can be used to guarantee a prescribed tracking accuracy. We apply the new method to the benchmark truck backer-upper problem. In sharp contrast to results from the literature [2], [11], we achieve acceptable tracking accuracy with a relatively simple controller and with minimal requirements for measured input-output data.

An interesting issue, not addressed in this paper, is the effect of using other membership function types on the design of the TSK fuzzy controller. Different membership function shapes (triangular–trapezoidal–Gaussian) generate different input-output plots (ellipses–circles–parabolas). Future work will extend the results presented here to other membership function shapes and their combinations.

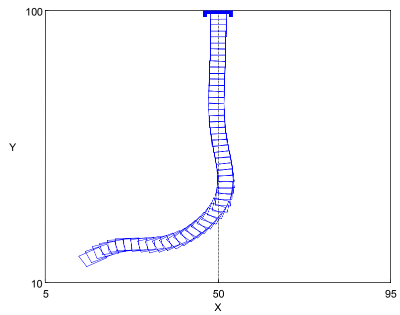

Figure 3 *Sample Truck trajectory for the human driver with initial position* (x,y,ϕ): (20,20,30).

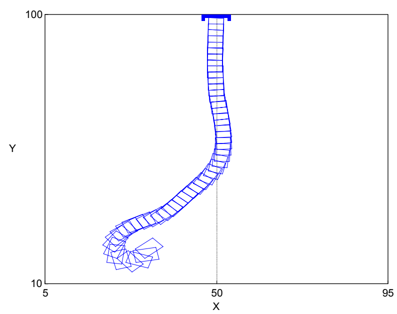

Figure 4 *Sample Truck trajectory for the human driver with initial position* (x,y,ϕ): (30,20,220).

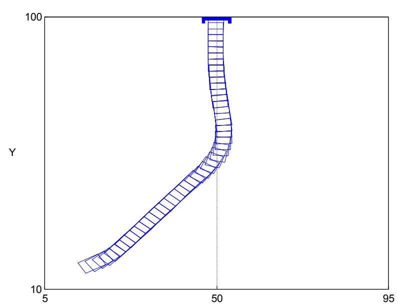

Figure 5 *Sample Truck trajectory of the fuzzy system for initial position* (x,y,ϕ): (20,20,30).

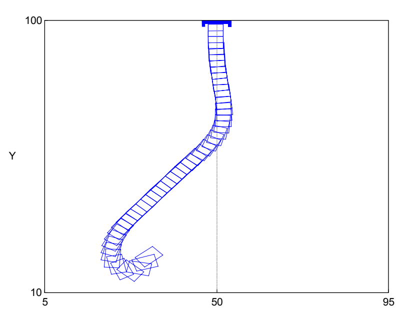

Figure 6 *Sample Truck trajectory for the fuzzy system with initial position* (x,y,ϕ): (30,20,220).

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