Nyquist Stability Criterion

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Why use the Nyquist Criterion?

Answers the questions:

Q1. Are there any poles in the RHP?

Q2. If the answer to Q1 is yes, then how many?

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Closed-loop Characteristic Equation

$$1+G(s)H(s)=1+L(s)=0$$

$$F(s)=1+L(s)$$

$$=1+\frac{N_L(s)}{D_L(s)}=\frac{N_L(s)+D_L(s)}{D_L(s)}$$

Zeros of $F(s)$ are closed-loop poles
Poles of $F(s)$ are open-loop poles

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Contour Mapping

- Contour = closed directed simple (does not cross itself) curve .
- Consider the angle change for $F(s)$ as s traverses a known contour D.
- Change in $\angle F$ = *sum of angle changes for its zeros − sum of angle changes of its poles.*
- If D is the Nyquist contour then the net angle change can be used to determine stability.

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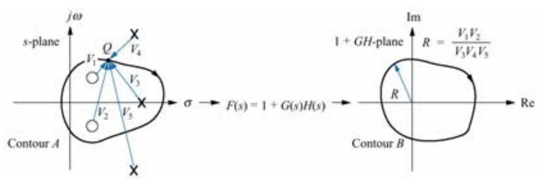
Angle Change for $F(s)$



360°(No. of closed-loop poles inside contour)
–360 °(No. of open-loop poles inside contour)

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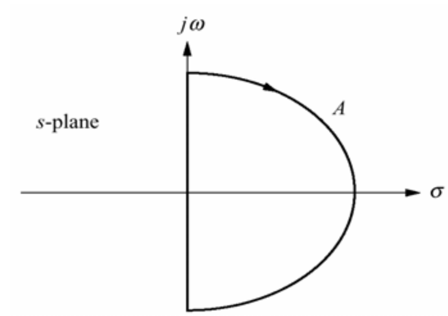
Nyquist Contour



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Mapping of $L(s)$

- $L(j\omega)$ = polar plot.
- $L(-j\omega)$ = complex conjugate of polar plot
  = mirror image of polar plot
  Assume real coefficients
- $L(s)$, large $|s|$,
  $|L(s)| \approx 0$, $\deg[N_L] < \deg[D_L]$
  $|L(s)|$ = constant, $\deg[N_L] = \deg[D_L]$

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Principle of the Argument

- If $F(s) = 1 + L(s)$ has Z zeros and P poles inside the Nyquist contour, a plot of $F(s)$ as s travels once (clockwise) around the contour encircles the origin of the complex plane in which it is plotted $(-N)$ times where $(-N) = Z - P$ = no. of clockwise encirclements

i.e. $N = P - Z$
= no. of counterclockwise encirclements

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Stability Results

(i) For stability (no closed-loop poles in the RHP)

Z=0 i.e. $N = P$

For an open-loop stable system (P = 0), N = 0

(ii) For an unstable system ($Z \neq 0$)

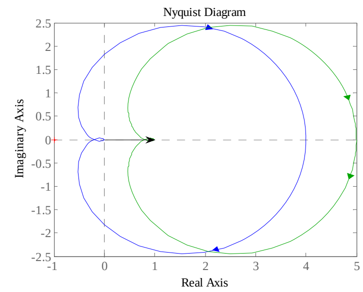
$Z = P - N$

= number of closed-loop poles in the RHP

(iii) The number of encirclements of the origin by $F(s) = 1 + L(s)$ is equal to the number of encirclements of (–1,0) by $L(s)$

Nyquist Plot

$F(s) = 1 + L(s)$ and $L(s)$



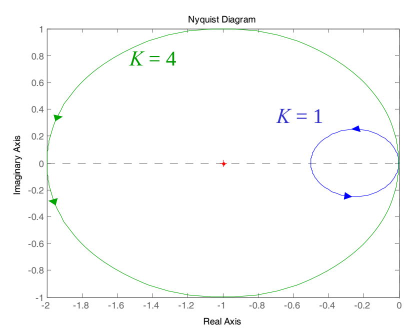
Nyquist Stability Criterion

1. **Necessary and sufficient** condition for the closed-loop stability of loop gain $L(s)$ is: $N = P$,
   N = no. of counterclockwise encirclements of (–1,0) by $L(s)$, P = no. of open-loop poles of $L(s)$
2. **For an unstable system**, the number of closed-loop system poles in the RHP = $Z = P - N$
3. **For open-loop stable systems** (P = 0),
   (a) the system is stable with no (–1,0) encirclements, and (b) the number of RHP poles for an unstable system = – N = number of clockwise encirclements of (–1,0)

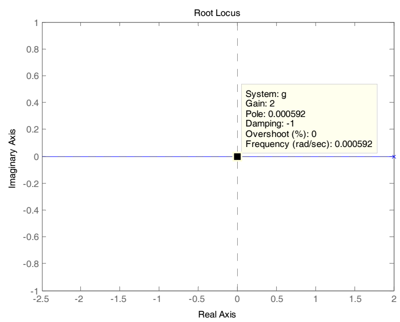
Open-loop Unstable System

$L(s) = K/(s-2)$ $\quad K = 1: \quad N = 0 \quad P = 1 \quad Z = 1$

$K = 4: \quad N = 1 \quad P = 1 \quad Z = 0$



Root Locus: Open-loop Unstable System



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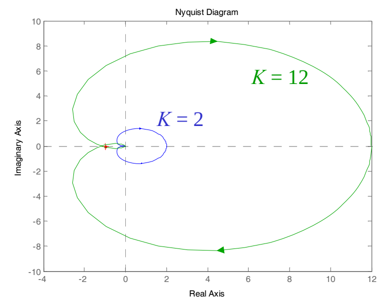
Nyquist Plot: 3rd Order, Type 0

$$L(s) = \frac{K}{(s+1)(s/2+1)(s/3+1)}$$

$K = 2: \quad N = 0 \quad P = 0 \quad Z = 0$

$K = 12: \quad N = -2 \quad P = 0 \quad Z = 2$



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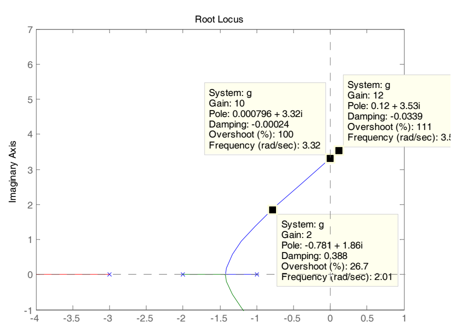
Root Locus: 3rd Order, Type 0



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Simplified Nyquist Criterion

- Assume that the loop gain function has no open-loop poles in the RHP (i.e. $P = 0$).
- Let an observer follow the polar plot of the loop gain in the direction of increasing frequency. The closed-loop system is stable **if and only if** the point (–1,0) is to **the left of the observer**.
- **Remark:** The criterion implies no encirclements of (–1,0) by the complete Nyquist plot.

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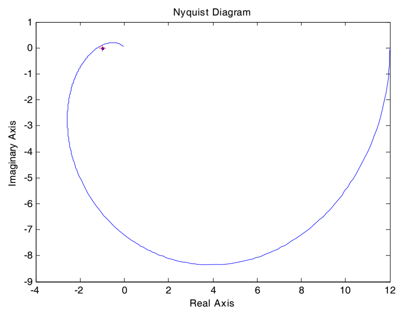
Example: Unstable System

$$L(s) = \frac{12}{(s+1)(s/2+1)(s/3+1)}$$



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Modified Nyquist Contour

- Add a small semicircle around the origin for systems of type ≥ 1
- The small semicircle is mapped to l infinite semicircles traversed clockwise for a type l system.

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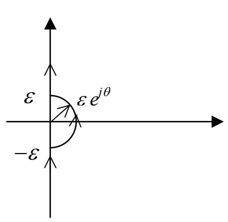
Large Semicircles

For small $|s| = \varepsilon$

$$G(s) = K_e \frac{\prod_{k=1}^{m} \frac{s}{\omega_k} + 1}{(s)^l \prod_{i=1}^{n-l} \frac{s}{\omega_i} + 1} \cong \frac{K_e}{(s)^l}$$



Net rotation for $s = +\pi$
Denominator angle $= l\ \pi$
Transfer function angle $= -\ l\ \pi$ (clockwise)

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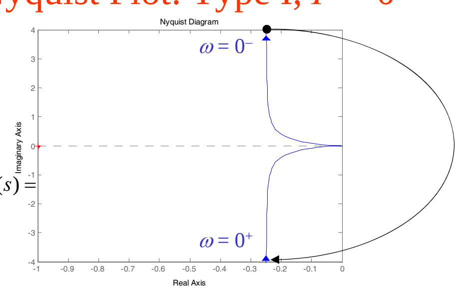
Nyquist Plot: Type I, $P = 0$

$L(s) =$



Type I: one clockwise half circle

$N = 0 \quad P = 0 \quad Z = 0$

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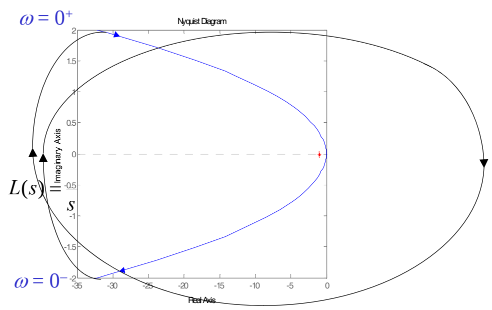
Nyquist Plot: Type II, $P = 0$



Type II: two clockwise half circles

$-N = 2 \quad P = 0 \quad Z = 2$

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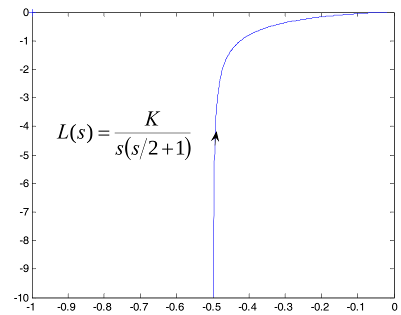
Simplified Nyquist: Type I



$N = 0$
Stable

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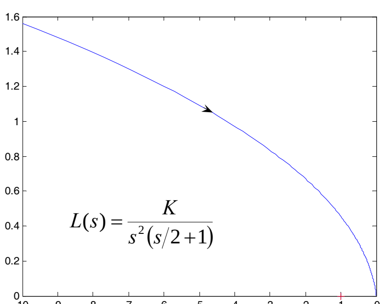
Simplified Nyquist: Type II



$N \neq 0$
Unstable

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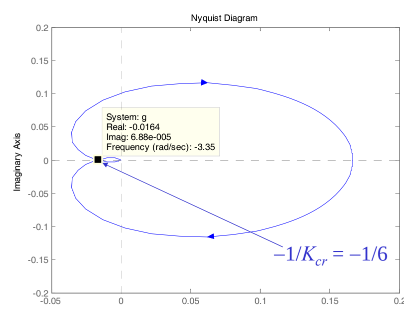
Nyquist Plot: Variable K

- Plot frequency response for a gain $K = 1$.
- Count encirclements of the point $-1/K$.



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